PLAY WITH MATH

TEST NO-06 TIME:-3Hrs.

F.M:-100

General instructions:-

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1-4 in section A are short-answer type questions carrying 1 mark each.
4. Questions 5-12 in section B are short-answer type questions carrying 2 marks each.
5. Questions 13-23 in section C are long-answer type questions carrying 4 marks each.
6. Questions 24-29 in section D are long-answer type questions carrying 6marks each.

Section-A

1 Find a vector in the direction of a vector $\vec{a} = \hat{\imath} - \hat{\jmath} + k$, which has magnitude 8 units.

2. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, find $|3AB|$.

3. If $x = at^2$ and $y = 2at$, find $\frac{d^2y}{dx^2}$.

4. Evaluate $\int \frac{logx}{x^2}$ dx

Or,

Find the differential equation of the family of all straight lines passing through the origin.

Section-B

5. If A and B are square matrices of same order and B is skew – symmetric matrix , show that A′BA is a skew-symmetric matrix.

6.Find the point of the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of abscissa.

Or,

Show that the function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ is not differential at x=2

7. Write the solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$.

8. find the local maxima and local minima of the function f(x) =(sinx – cosx),where $0<x<2\pi$.

9. If the point (-1,-1 ,2),(2,m,5) and (3,11,6) are collinear ,then find the value of m.

10. One card is drawn at random from a pack of well-shuffled deck of cards.

Let E : the card drawn is spade , F: the card drawn is an ace .

Are the events E and F independent ?

11. A company manufactures two type of sweaters type A and type B. it costs ₹360 to make a type A sweaters and ₹120 to make a type B sweaters. The company can make almost 300 sweaters and spend almost ₹ 72000 a day. The numbers of sweaters of type B cannot exceed the numbers of sweaters of type A by more than 100. The company make profit of ₹ 200 for each sweater type A and ₹120 for every sweater of type B.

Formulate this problem as a lpp to maximize the profit to the company.

12. Evaluate $\int_0^4 |x-1|\,dx$.

Section –C

13.If $\tan^{-1}(\frac{x-2}{x-4})+\tan^{-1}(\frac{x+2}{x+4}) = \frac{\pi}{4}$, then find the value f x.

14. Show that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)\,(a^2+b^2+c^2)$.

Or,

If f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ by using properties of determinants, find the value of f(2x)-f(x) .

15. Differentiates $\cos^{-1}(\frac{1-x^2}{1+x^2})$ w.r.t $\tan^{-1}(\frac{3x-x^3}{1-3x^2})$.

Or,

If $y = \frac{\sin^{-x}}{\sqrt{1-x^2}}$ show that $(1-x)^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

16. Evaluate $\int \frac{dx}{\sin^4 x + \cos^4 x}$

17. Evaluate $\int_0^{\pi} \frac{dx}{3+2\sin x+\cos x}$

Or,

. Evaluate $\int_{-5}^{0} f(x)dx$ where f(x) =|x| +|x+2| +|x+5|.

18 Find the solution of differential equation

$x^2dy + y(x + y)\,dx = 0$ if x=1 and y =1

19. Show that area of parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$,Also ,find the area of the parallelogram whose diagonals are $2\vec{\imath} - \vec{j}+\vec{k}$ and $\vec{\imath} + 3\vec{j} - \vec{k}$.

20. $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero non-coplanar vectors. Prove that $\vec{a}$-$2\vec{b}$ $+3\vec{c}$, $-2\vec{a}+3\vec{b}$ -$4\vec{c}$ and $\vec{a}$-$3\vec{b}$ $+5\vec{c}$ are coplanar vector.

21. A clever student used a biased coin so that the head is 3 times as likely to occur as tail . if the coin is tossed twice, find the probability distribution and mean of numbers of tails.

22 An urn contains m white and n black ball . A ball is drawn at random and is put back into the urn along with' k' additional ball of the same color as that of the ball drawn. A ball is again drawn at random.so that the probability of drawing a white ball does not depend on 'k' .

23 A librarian has to accommodate two different types of books on a shelf . The books are 6cm and 4 cm think and weight 1kg and $1\frac{1}{2}$ kg each, respectively . The shelf is 96 cm long and atmost can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? make it as an lpp and solve it graphically. What are the values reflected in the question ?

Section-D

24. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, Find AB use this to solve the following system of equations x-y = 3 , $2x + 3y + 3z + 4z = 17$ and $y+2z = 7$.

25. A jet of enemy country is flying along the curve $x^2 = 4y$. A solider placed at point (-1,2) want to shoot down the jet f enemy when it is nearest to him. Find the nearest point to the solider. How does this problem help solider in the battle field ? justify your answer.

26. A variable plane which remains at a constant distance 3p from the origin cut then coordinate axes at A,B and C show that the locus of the centroid of ΔABC is

$x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

or,

Find the vector equation of the line passing through (1,2,3) and parallel to the planes $\vec{r}.(\vec{\imath}-\vec{\jmath}+2\vec{k})=5$ and $\vec{r}.(3\vec{\imath}+\vec{\jmath}+\vec{k})=6$.

27. If f:R-{2}→R –{3} is defined by f(x) = $\frac{3x+1}{x-2}$ where R is set of real numbers, show that f is invertible and hence find the value of f^{-1}

Or,

Let f:N→R be a function defined as $f(x) = 4x^2 + 12x + 15$, show that f:N→range(f) is invertible. Find the inverse of f.

28. Find the area of region bounded by the curve $y^2 = 4ax$ and $x^2 = 4ay$

Or,

Using integration, find the area of triangular region whose side have the equation

$y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

29. solve the differential equation

$(x+2y^2)\frac{dy}{dx} = y$,given that when x = 2 and y =1 . if x denotes the percentage of people who are polite and y denotes the percentage of people who are intelligent. Find x, when y =2% . A polite person is always liked by all in society. Do you agree justify.